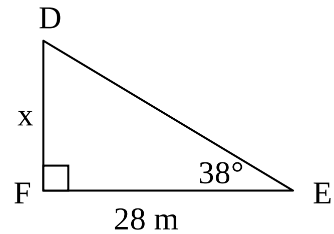
Right Triangles and SOHCAHTOA: Finding the Length of a Side Given One Side and One Angle

Preliminary Information: "SOH CAH TOA" is an acronym to represent the following three trigonometric ratios or formulas:

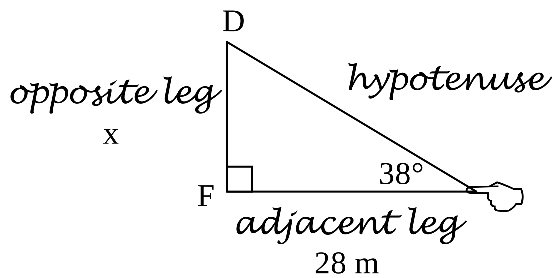
$$\sin\theta = \frac{\text{opposite leg}}{\text{hypotenuse}}\ ;\ \cos\theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}\ ;\ \tan\theta = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

Part I) Model Problems

Example 1: Consider right ΔDEF pictured at right. We know one acute angle and one side, and our goal is to determine the length of the unknown side x.



Step 1: Place your finger on the 38° angle (the acute angle), and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg. (The word "adjacent" usually means "next to.")



Step 2: We need to determine which trigonometric ratio to use: the sine, the cosine, or tangent. It is recommended that you write "SOH CAH TOA" on your paper:

SOH CAH TOA

Step 3: Ask yourself, "Which side do I know?" In other words, which side has a length we already know? In this example, we know that one side is 28 m, so we know the adjacent leg. Underline both of the A's in SOH CAH TOA to indicate that we know the **A**djacent leg:

SOH C<u>A</u>H TO<u>A</u>

Step 4: Now ask yourself, "Which side do I want to find out?" In other words, which side length are we being asked to calculate? In this example, we are being asked to calculate the side marked x, so we want the opposite leg. Underline both of the O's in SOH CAH TOA to indicate that we want the **O**pposite leg:

$$S\underline{O}H\ C\underline{A}H\ T\underline{OA}$$

Step 5: Consider which of the three ratios has the most information: we have one piece of information for the sine (one underline), only one piece of information for the cosine (one underline), yet we have two pieces of information for the tangent (two underlines). We are therefore going to use the tangent ratio formula:

$$\tan\theta = \frac{opposite\ leg}{adjacent\ leg}$$

Step 6: Substitute the known information into the formula:

$$\tan\theta = \frac{opposite\ leg}{adjacent\ leg} \Rightarrow \tan 38^\circ = \frac{x}{28}$$

(Note that we dropped the units of "meters" for simplicity; the answer will be in meters.)

Step 7: Solve for x. In this example, it is probably simplest to multiply both sides by 28:

$$\tan 38^\circ = \frac{x}{28}$$
$$28 \cdot \tan 38^\circ = 28 \cdot \frac{x}{28}$$
$$x = 28 \cdot \tan 38^\circ$$

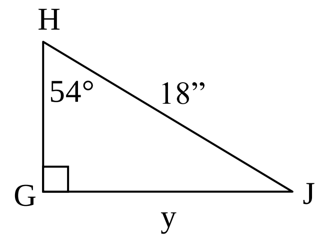
Step 8: Simplify. You may use a handheld calculator (in degrees mode), on online Sine Cosine Tangent Calculator, or a table of values from a chart. In this case, an approximate value for the tangent of 38 degrees is 0.78129:

$$x = 28(0.78129)$$
$$x = 21.876m$$

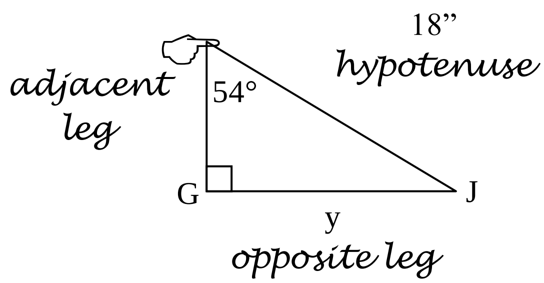
(Note that we have included units of meters, as the original side was specified in meters.)

Step 9: Check for reasonableness: In this case, the acute angle was 38°, which is less than 45°. (If it had been a 45° angle, both legs would be congruent.) It is reasonable that this leg should be less than 28m. ☺

Example 2: Consider right ΔGHJ pictured at right. We know one acute angle and one side, and our goal is to determine the length of the unknown side y to the nearest inch.



Step 1: Place your finger on the 54° angle (the acute angle), and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg.



Step 2: We need to determine which trigonometric ratio to use: the sine, the cosine, or tangent. It is recommended that you write “SOH CAH TOA” on your paper:

SOH CAH TOA

Step 3: Ask yourself, “Which side do I know?” In this example, we know that the hypotenuse is 18 inches. Underline both of the H’s in SOH CAH TOA:

$$SO\underline{H}\ \ CA\underline{H}\ \ TOA$$

Step 4: Now ask yourself, “Which side do I want to find out?” In this example, we are being asked to calculate the side marked y, so we want the opposite leg. Underline both of the O’s in SOH CAH TOA:

$$S\underline{OH}\ \ CA\underline{H}\ \ T\underline{O}A$$

Step 5: Consider which of the three ratios has the most information: we have two pieces of information for the sine:

$$\sin\theta = \frac{opposite\ leg}{hypotenuse}$$

Step 6: Substitute the known information into the formula:

$$\sin\theta = \frac{opposite\ leg}{hypotenuse} \Rightarrow \sin 54° = \frac{y}{18}$$

(Note that we dropped the units of "inches" for simplicity.)

Step 7: Solve for y. In this example, it is probably simplest to multiply both sides by 18:

$$\sin 54° = \frac{y}{18}$$

$$18 \cdot \sin 54° = 18 \cdot \frac{y}{18}$$

$$y = 18 \cdot \sin 54°$$

Step 8: Simplify. In this case, an approximate value for the sine of 54 degrees is 0.80902:

$$y = 18(0.80902)$$

$$y = 14.5623''$$

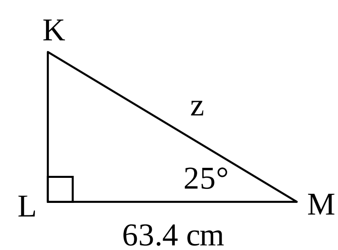
To the nearest inch, we get $y = 15''$

(Note that we have included inches.)

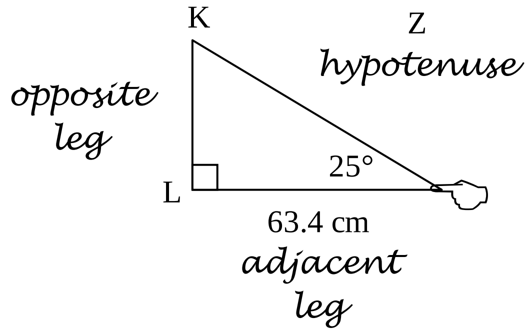
Step 9: Check for reasonableness: In this case, the hypotenuse must be longest at 18 inches, so a leg of 15" seems reasonable. ☺

Example 3: (Note: This example is generally more difficult for students to complete correctly due to a significant change in the algebra required: we will end up with an equation in which the variable is in the denominator of a fraction, and the algebra steps required are different.)

Consider right ΔKLM pictured at right. We know one acute angle and one side, and our goal is to determine the length of the unknown side marked z to the nearest tenth of a centimeter.



Step 1: Place your finger on the acute angle, and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg.



Step 2: We need to determine which trigonometric ratio to use: the sine, the cosine, or tangent. It is recommended that you write "SOH CAH TOA" on your paper:

SOH CAH TOA

Step 3: Ask yourself, "Which side do I know?" In this example, we know that the adjacent leg is 63.4 cm. Underline both of the A's in SOH CAH TOA:

SOH C<u>A</u>H TO<u>A</u>

Step 4: Now ask yourself, "Which side do I want to find out?" In this example, we are being asked to calculate the side marked z, the hypotenuse. Underline both of the H's in SOH CAH TOA:

SO<u>H</u> C<u>AH</u> TO<u>A</u>

Step 5: Consider which of the three ratios has the most information: we have two pieces of information for the cosine:

$$\cos\theta = \frac{adjacent\ leg}{hypotenuse}$$

Step 6: Substitute the known information into the formula:

$$\cos\theta = \frac{adjacent\ leg}{hypotenuse} \Rightarrow \cos 25° = \frac{63.4}{z}$$

(Note that we dropped the units of "centimeters" for simplicity.)

Step 7: Solve for the variable. In this example, note that the variable is in the denominator of the expression, so we cannot multiply both sides of the equation by 63.4: Instead, we need a different approach. Two of the most common techniques are shown below. Both are correct.

Method 1: Multiply both sides by the denominator	Method 2: Cross-multiply
$\cos 25° = \frac{63.4}{z}$ $z \cdot \cos 25° = z \cdot \frac{63.4}{z}$ $z \cdot \cos 25° = 63.4$	$\cos 25° = \frac{63.4}{z}$ $rewrite\ as\ a\ \ fraction:$ $\frac{\cos 25°}{1} = \frac{63.4}{z}$ $z \cdot \cos 25° = 63.4$

We now can get z by itself by dividing both sides by $\cos 25°$:

$$z \cdot \cos 25° = 63.4$$
$$\frac{z \cdot \cos 25°}{\cos 25°} = \frac{63.4}{\cos 25°}$$
$$z = \frac{63.4}{\cos 25°}$$

Step 8: Simplify. The approximate value for the cosine of 25 degrees is 0.90631:

$$z = \frac{63.4}{\cos 25°}$$
$$z = \frac{63.4}{0.90631}$$
$$y = 69.9542$$

To the nearest tenth of a centimeter, we get $z = 70.0cm$

(Note that we have included centimeters.)

Step 9: Check for reasonableness: In this case, the hypotenuse must be longest, and 70.0 cm is greater than 63.4 cm, so it seems reasonable. ☺

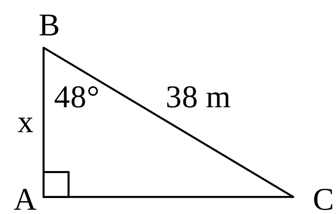
Part II) Practice Problems

1. Calculate the value of x to the nearest tenth: $\sin 38° = \dfrac{x}{80}$

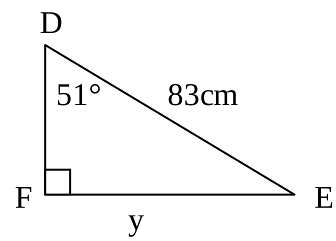
2. Calculate the value of y to the nearest tenth: $\cos 52° = \dfrac{y}{80}$

3. Calculate the value of z to the nearest hundredth: $\tan 24° = \dfrac{z}{34.627}$

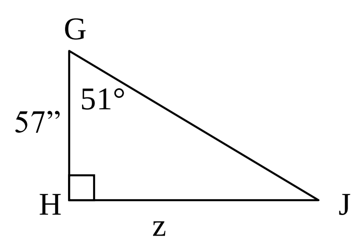
4. Determine the length of side x to the nearest tenth.



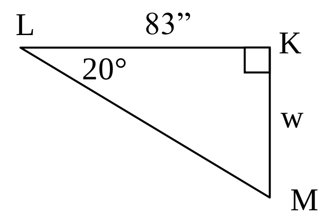
5. Determine the length of side y to the nearest hundredth.



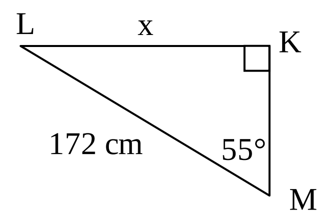
6. Determine the length of side z to the nearest inch.



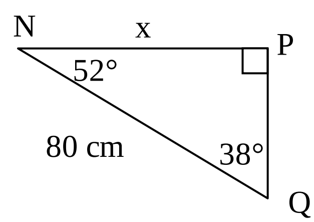
7. Determine the length of side w to the nearest inch.



8. Determine the length of side x to the nearest hundredth.



9. For the triangle pictured, Marcy placed her finger on the 38° angle and concluded that $\sin 38° = \frac{x}{80}$. Likewise, Timmy placed his finger on the 52° angle and concluded that $\cos 52° = \frac{x}{80}$.



a) If you solve it Marcy's way, what answer will she get?

b) If you solve it Timmy's way, what answer will he get?

c) Are these results reasonable? Explain.

Part III) Challenge Problems

10. As we saw in problem 9, there is a connection between $\sin 38°$ and $\cos 52°$.

a) How are the angles 38° and 52° geometrically related? (Think back to what you know about angles from Geometry.)

b) Make a conjecture based on problems 9 and 10a: The sine of 20° must be equal to the cosine of ______° because the two angles ____________________.

c) State your conjecture as a formula: $\sin\theta =$ _____

d) Verify that your formula works correctly for $\theta = 37°$.

11. Error Analysis: Consider the following equation: $\tan 24° = \dfrac{34.627}{z}$

a) Calculate the value of z to the nearest hundredth.

b) Substitute your answer for z into the expression $\dfrac{34.627}{z}$ and show that it really is the same as $\tan 24°$.

c) If your answers match, move on to the next problem. If your answers don't match, you probably multiplied both sides of the equation in part (a) by 34.627. Redo the problem by multiplying both sides by z or by using cross-multiplication. It may help to refer back to example 3.

12. Consider the equation $\tan 74° = \dfrac{x}{58cm}$

a) Sketch and label a right triangle that matches this equation.

b) Solve for x. Round to the nearest hundredth.

c) Determine the hypotenuse of your triangle. Round to the nearest hundredth.

d) Use the Pythagorean Theorem to confirm that this is, in fact, a right triangle.

13. Consider the following information: In ΔABC with right $\angle C$, the measure of $\angle A = 31°$. The length of side AB is 42cm.

a) Sketch and label a right triangle that matches this description.

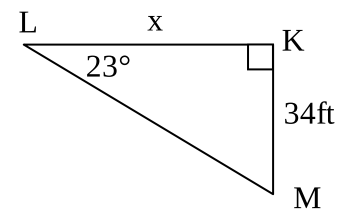
b) Determine the length of side BC.

c) Determine the length of the third side.

(continued on next page)

14. Error Analysis: Consider the right triangle pictured at right, which Camryn and Isabel are both trying to solve. They both set it up using the equation $\tan 23° = \dfrac{34}{x}$

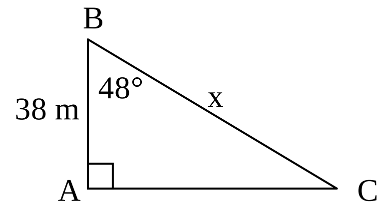
The steps of their work is shown below. Analyze their work and determine who, if anyone, is doing it correctly.



Camryn's work	Isabel's work
$\tan 23° = \dfrac{34}{x}$ $34 \cdot \tan 23° = 34 \cdot \dfrac{34}{x}$ $34 \tan 23° = x$ $x = 14.43$	$\tan 23° = \dfrac{34}{x}$ $\textit{rewrite over } 1:$ $\dfrac{\tan 23°}{1} = \dfrac{34}{x}$ $\textit{cross} - \textit{multiply}:$ $x \cdot \tan 23° = 34$ $\dfrac{x \cdot \tan 23°}{\tan 23°} = \dfrac{34}{\tan 23°}$ $x = \dfrac{34}{\tan 23°}$ $x = 80.10$

15. Consider the triangle at right:

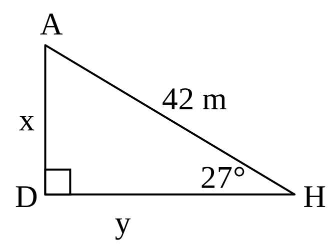
a) Determine the length of side x to the nearest tenth.



b) Is side x, the hypotenuse, actually longer than 38 m? If not, find your error.

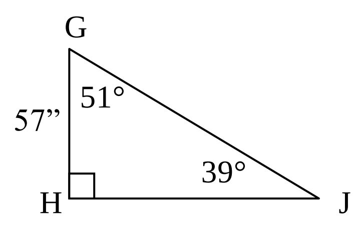
16. Answer the following questions about ΔDAH :

a) How long is side **x**? [Hint: Ignore side y. Just pretend it's erased for a minute.]



b) How long is side **y**? [Hint: Ignore side x – just pretend it got erased for a minute.]

17. Determine the perimeter of the following triangle:



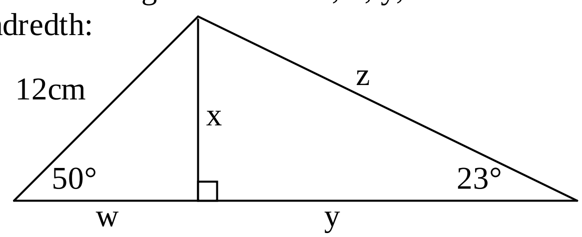
18. A 32-foot ladder is leaning against a tree. The ladder forms a 72° angle with the ground, not the tree. Assuming the tree is growing straight up:

a) Make a labeled sketch of the situation.

b) How high up the tree does the ladder reach?

c) How far away from the tree is the base of the ladder?

19. Determine the lengths of sides w, x, y, and z in the figure. Round answers to the nearest hundredth:

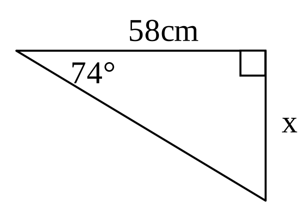


Part IV) Answer Key

1. 49.3
2. 49.3
3. 15.42
4. 25.4 m
5. 64.50 cm
6. 70”
7. 30”
8. 140.89 cm
9. a) 49.3 cm
   b) 49.3 cm
   c) These results match, which is reasonable, because it’s the same triangle and both are solving for the same side.
10. a) These two angles are complementary; their measures add up to 90°.
    b) The sine of 20° must be equal to the cosine of 70° because the two angles are complementary (or their measures add up to 90°).
    c) $\sin\theta = \cos(90° - \theta)$
    d)
    $$\sin 37° = \cos(90° - 37°)$$
    $$\sin 37° = \cos(53°) \qquad \text{; Yes, it checks.}$$
    $$0.602 = 0.602$$
11. Consider the following equation: $\tan 24° = \dfrac{34.627}{z}$
    a) Correct answer is 77.77; (Note: the most common wrong answer is 15.42.)

    b)
    $$\tan 24° \stackrel{?}{=} \frac{34.627}{77.77}$$; yes, it matches. (Note: some will get $0.445 \neq 2.25$)
    $$0.445 \stackrel{?}{=} 0.445$$
    c) Students who found it matched move on; those who didn’t should go back and multiply both sides by z or cross-multiply instead of multiplying both sides by 34.627.

12. a) One possible sketch is shown at right:



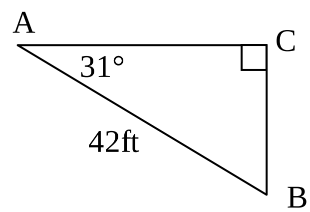
    b) 202.27 cm

    c) 210.42 cm

    d)

$$58^2 + 202.27^2 = 210.42^2$$
$$3364 + 40913.1529 = 44276.5764 \text{; This is within roundoff error.}$$
$$44277.1529 = 44276.5764$$

13. a) One possible sketch is shown at right:
b) 21.63 cm
c) 36.00 cm



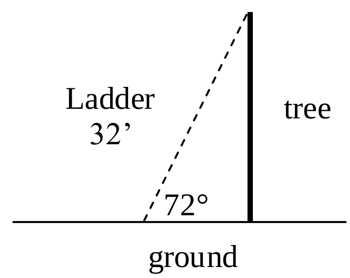
14. Camryn is incorrectly multiplying both sides by the numerator; Isabel's procedure is correct.

15. a) 56.8 m (Note: the most common wrong answer is 25.4 m).
b) Yes, the hypotenuse is longer than the leg.

16. a) 19.07 m
b) 37.42 m

17. 217.96"

18. a) One possible ketch is shown at right.
b) 30.43 feet
c) 9.89 feet



19. w = 7.71 cm
x = 9.19 cm
y = 21.66 cm
z = 23.53 cm